Theory of machine

If you have a smart project, you can say "I'm an engineer"

Lecture 4

Instructor

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Theory of machine
MDP 234

- **Lecture aims:**
  - Understand the Acceleration diagram.
  - Identify the *Coriolis* Component of Acceleration

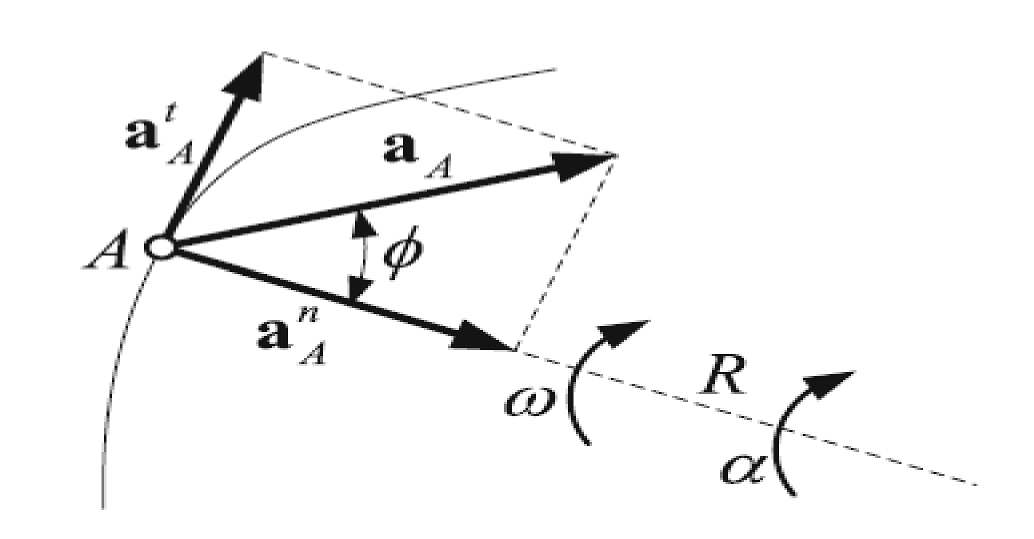
.

Acceleration of a Point

- The acceleration of a point is the relationship between the change of its velocity vector and time .

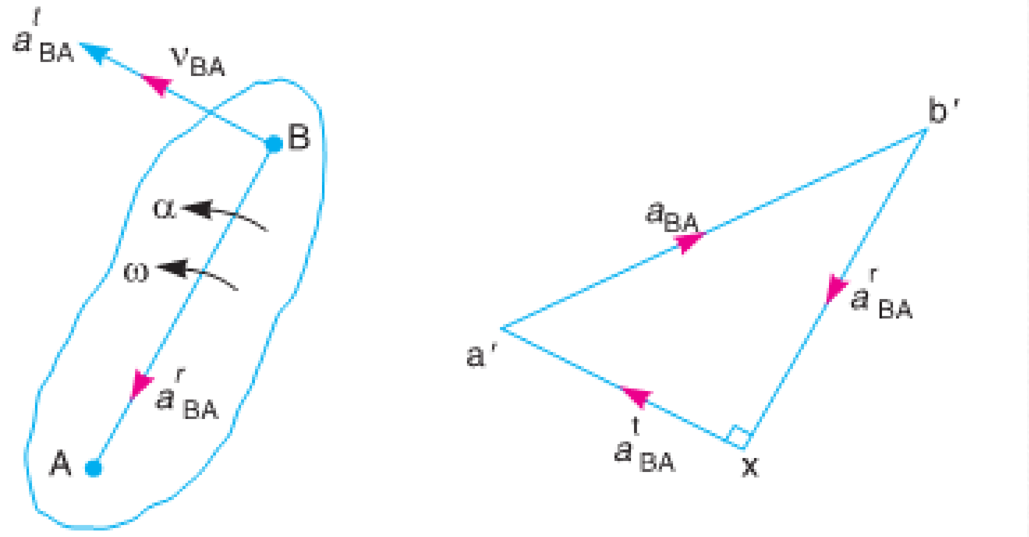
Acceleration of a Point

- The acceleration of a point has a normal component that points towards the center of the trajectory and a tangential component whose direction is tangential to the trajectory



Acceleration of a Link

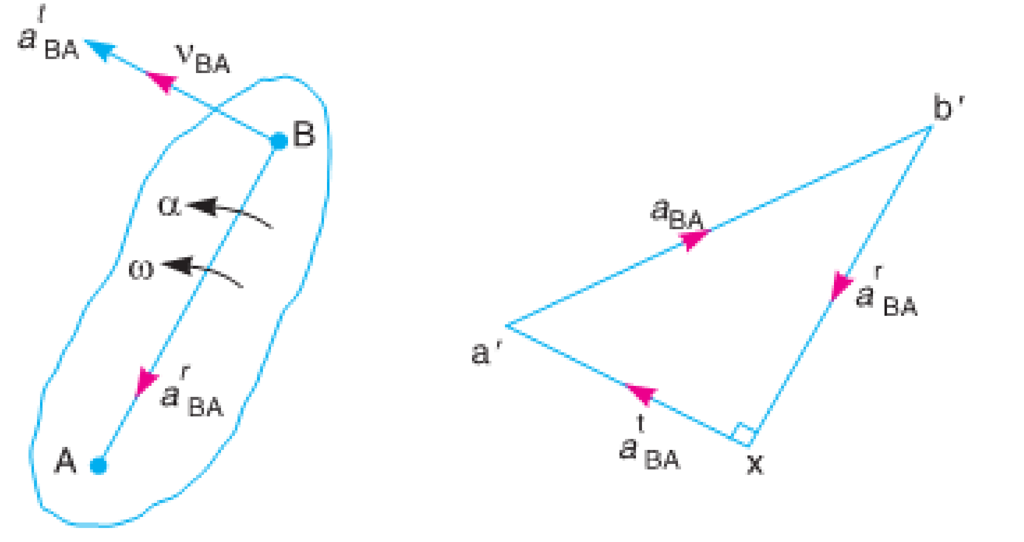
- Consider two points *A* and *B* on a rigid link as shown in Fig. 8.1 (*a*). Let the point *B* moves with respect to *A*, with an angular velocity of ω rad/s and let α rad/s2 be the angular acceleration of the link *AB*.



(*a*) Link. (*b*) Acceleration diagram.

Acceleration of a Link

- Since the point B moves with respect to A with an angular velocity of ω rad/s, therefore centripetal or radial component of the acceleration of B with respect to A ,
- This radial component of acceleration acts perpendicular to the velocity v_{BA}, In other words, it acts ***parallel* to the link *AB*.**



(*a*) Link. (*b*) Acceleration diagram.

$$a^r_{BA} = \omega^2 \times \text{Length of link } AB = \omega^2 \times AB = v_{BA}^2 / AB$$

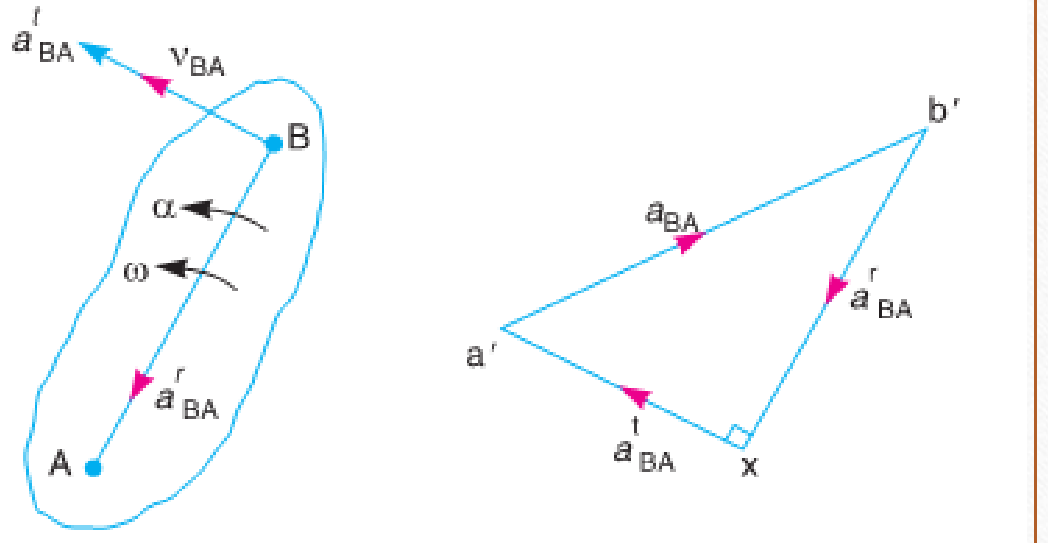
$$\left(\because \omega = \frac{v_{BA}}{AB} \right)$$

Acceleration of a Link

- We know that tangential component of the acceleration of B with respect to A ,
- This tangential component of acceleration acts parallel to the velocity v_{BA}. In other words, it acts ***perpendicular* to the link *A B*.**

$$a^t_{BA} = \alpha \times \text{Length of the link } AB = \alpha \times AB$$

- α = zero if the crank rotate with constant speed



(a) Link. (b) Acceleration diagram.

Acceleration diagram

Translated bodies

- ∵ the motion is absolute, then select any fixed point such as o be as a reference point (i.e point of zero acceleration).
- Draw the path of translation.
- If a_B is known, select a scale factor to draw the acceleration diagram (denoted by SFa)

$$SFa = \frac{draw\ value\ in\ mm}{actual\ value\ of\ acceleration\ in\ (m/s^2)} = \frac{ob}{a_B}$$

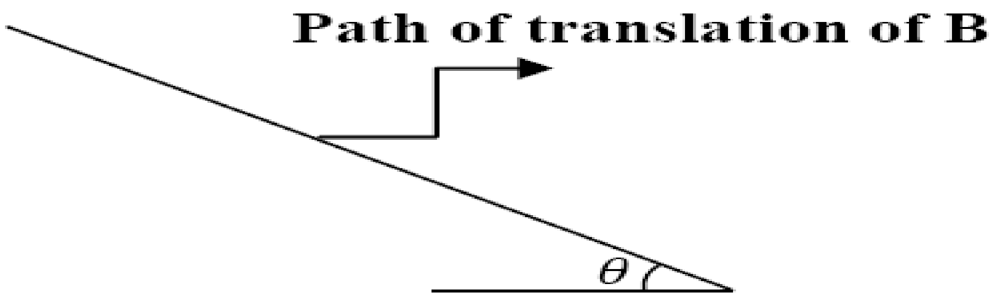
Acceleration diagram

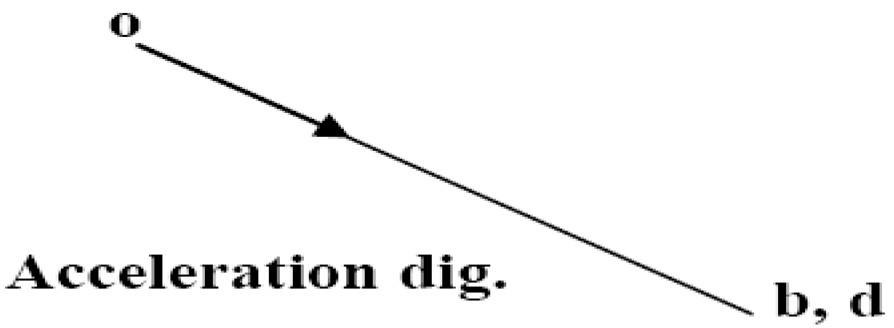
Translated bodies

In which ob=(a_B)(SFa).

Then all points on the piston have the same acceleration value.

Note: the base (ref.) point o of v_o =0, a_o=0.





Acceleration diagram

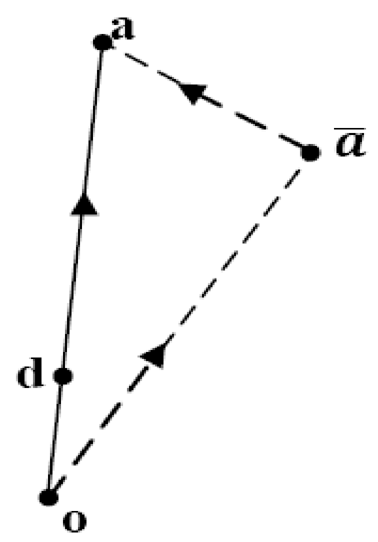
Bodies rotate about fixed point   α = Given

Also we have two method:-

1- If α is given:-

a_{An}=(oA) ω^2 ↗ normal component of acceleration of A relative to rotation centre.

a_{At}=(oA) α ↖ normal component of acceleration of A relative to rotation centre.



acceleration diagram

Acceleration diagram

Bodies rotate about fixed point **α = Given**

- Finally connect oa to represent the absolute value of acceleration of point A. $\Longrightarrow a_A = a_{Ao} = \frac{oa}{SFa}$.

  To find the acceleration of any point located on the link, such as point D. specify d on oa such that $\frac{od}{oa} = \frac{OD}{OA}$

  $\Longrightarrow od = \left(\frac{OD}{OA}\right) oa.$

Acceleration diagram

Bodies rotate about fixed point

- Select a reference point of zero acceleration (point o)
- Select $SFa = \frac{\text{drawn value of } a_{An} \text{ or } a_{At}}{\text{actual value of } a_{An} \text{ or } a_{At}}$. depend on which is greater a_{An} or a_{At}.
- Start from o to draw $o\bar{a}$ // OA directed into the rotation centre, by value of $o\bar{a} = a_{An} . SFa$.
- From point $\bar{a}$ draw $\bar{a}a \perp OA$ in direction of α by value $\bar{a}a = a_{At}.SFa$.

Acceleration diagram

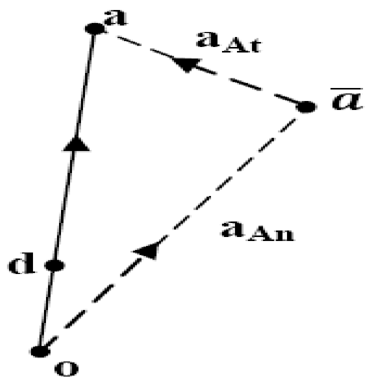
Bodies rotate about fixed point **α = Unknown**

- Start from O, draw two lines.
  - First line $o\bar{a} = a_{An} \,.SFa \,//\, OA$ directed in to point O.
  - Second line $oa = a_A \,.SFa$ in direction of a_A (given).

Then connect $a\bar{a}$ to represent the drawn tangential component of acceleration of A.

$$\Longrightarrow a_{At} = \frac{a\bar{a}}{SFa}$$

$$\alpha = \frac{a_{At}}{OA} \curvearrowright .$$



acceleration diagram

Acceleration diagram

Bodies under general plane motion

To draw the acceleration diagram it's required one of following:-

1- ω or V_{BC}.

- Absolute acceleration of any point (value **and** direction).
- Absolute acceleration of other point (value **or** direction).

2- ω or V_{BC}.

- Absolute acceleration of any point (value **and** direction).
- Angular acceleration of the link.

Acceleration diagram

Bodies under general plane motion

Steps:

- Find $a_{BCn} = \omega^2 BC = \frac{V_{BC}^2}{BC}$.
- If a_c is known (value and direction), V_B is known (direction).
- Select SFa $\Longrightarrow$ $oc = SFa.ac$, $c\grave{b} = SFa.a_{BCn}$.
- Start from point of zero acceleration such as o.
- Draw oc in direction of a_c.

Acceleration diagram

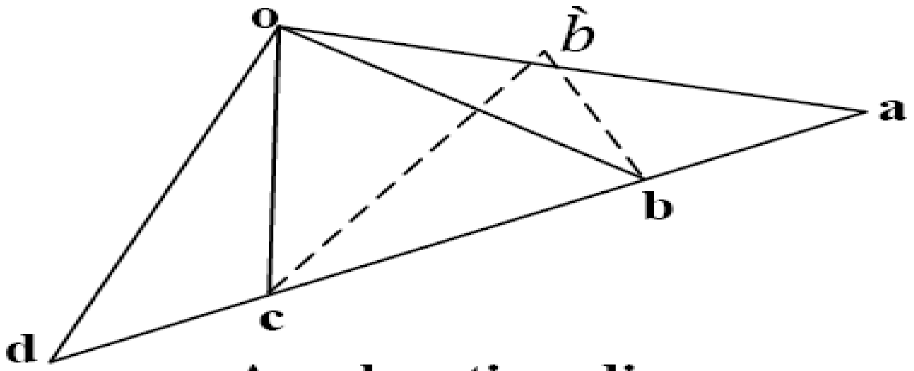
Bodies under general plane motion

- From c draw $c\grave{b}$ // link directed into point c (on the link).
- From o draw a line in direction of a_B, and from $\grave{b}$ draw a line ⊥ link (to be a_{BCt}), they are intersected at b.
- If α is known (value and direction).
- Find $a_{BCt} = \alpha . BC$ then $\grave{b}b = SFa . a_{BCt}$.

Acceleration diagram

Bodies under general plane motion

- Start from $\grave{b}$ to draw $\grave{b}b$ $\perp$link.
- Connect ob
- Find a_A , a_D.
- Specify bc such that $\frac{ba}{cb} = \frac{BA}{CB}$.
- Measure oa$\Longrightarrow a_A = \frac{oa}{SFa}$.
- Specify cd such that $\frac{ba}{cb} = \frac{BA}{CB}$.
- Measure $od \Longrightarrow a_D = \frac{cd}{SFa}$.

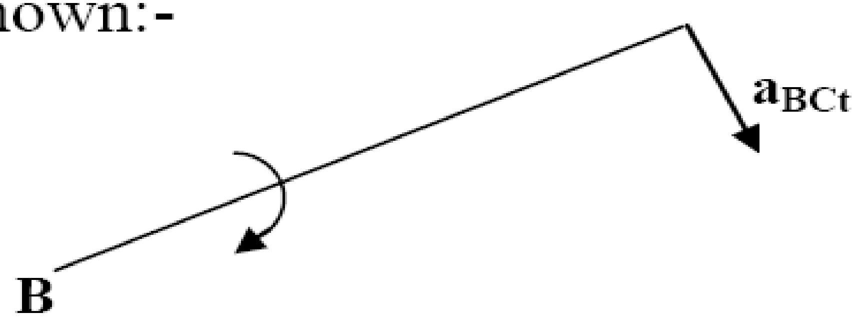


Acceleration diagram

Acceleration diagram

Bodies under general plane motion

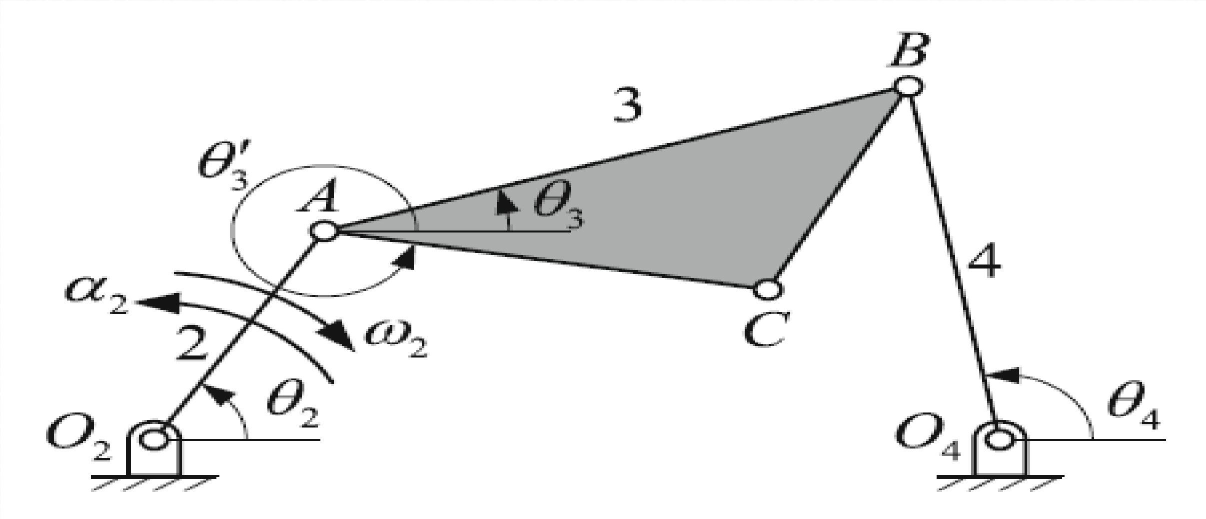
- To find α (value and direction) in unknown:-
- Measure $b\grave{b} \Rightarrow a_{BCt} = \frac{b\grave{b}}{SFa}$
- $\Rightarrow \alpha = \frac{a_{BCt}}{BC}$ ↺



Note:- α is the same for all points of the link.

Computing Acceleration in a Four-Bar Linkage

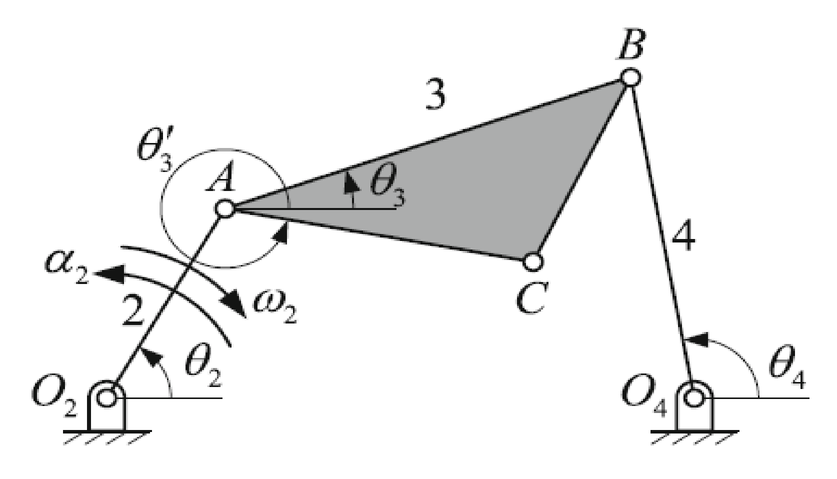
- Four-bar linkage with known angular velocity and acceleration of the input link .

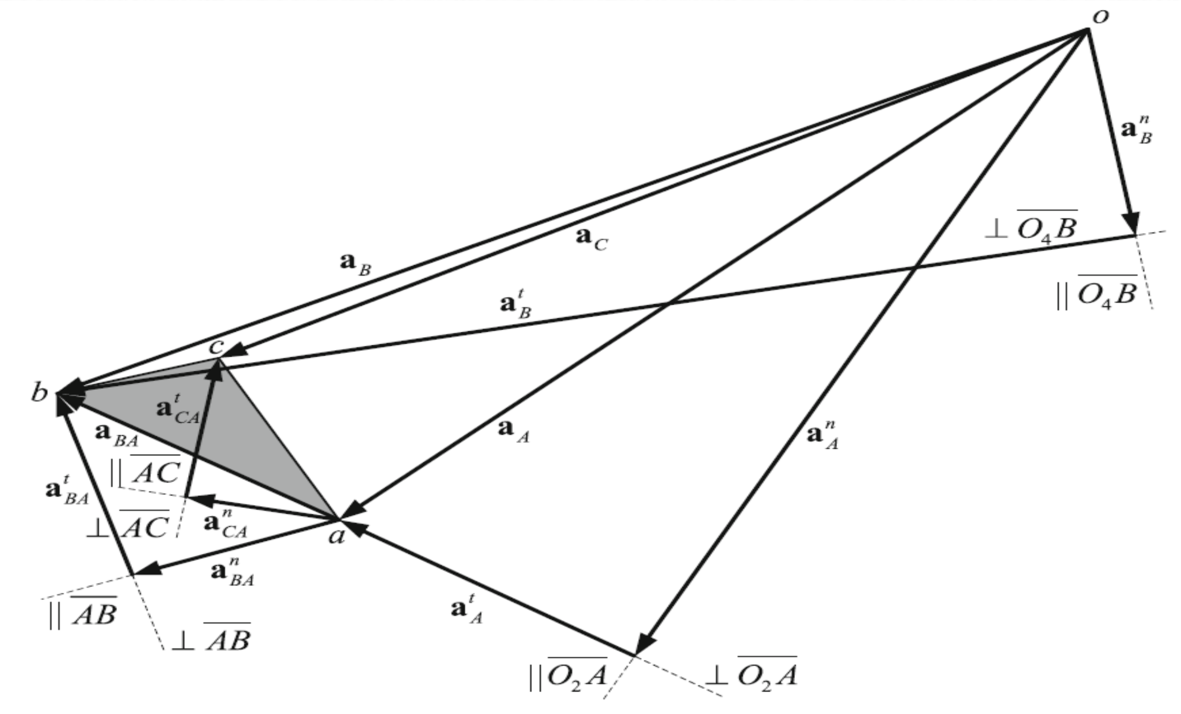


$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{BA} = (\mathbf{a}_B^n + \mathbf{a}_B^t) = (\mathbf{a}_A^n + \mathbf{a}_A^t) + (\mathbf{a}_{BA}^n + \mathbf{a}_{BA}^t)$$

Computing Acceleration in a Four-Bar Linkage

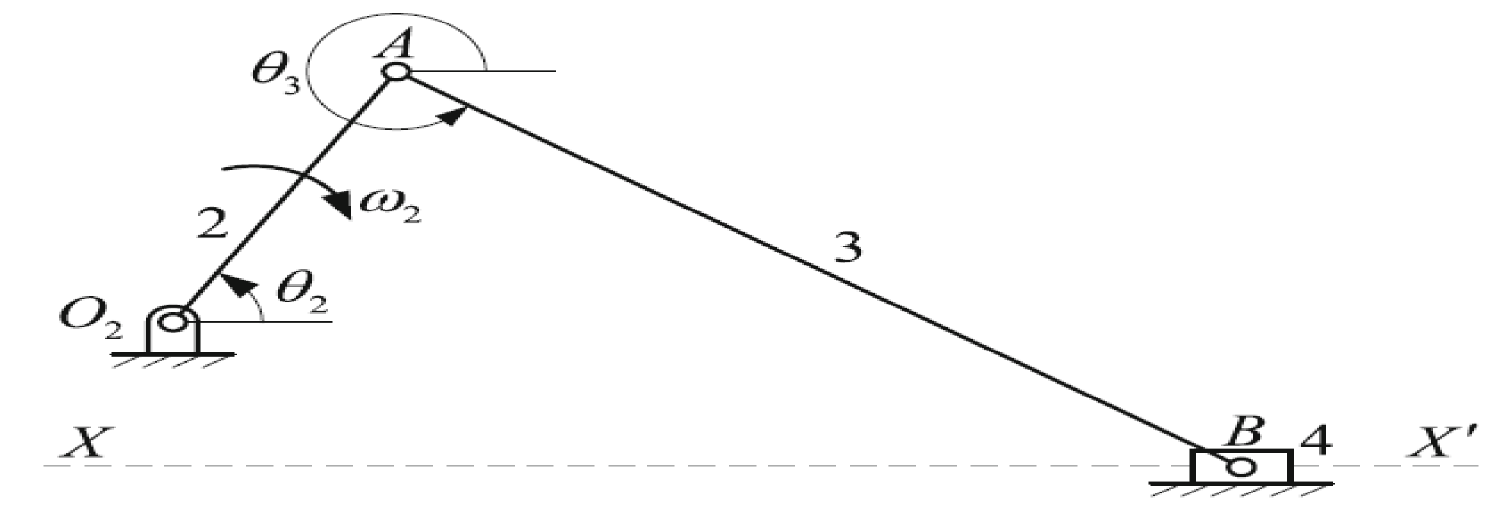
- Acceleration polygon of the four-bar linkage in the Fig.





Accelerations in a Slider-crank Linkage

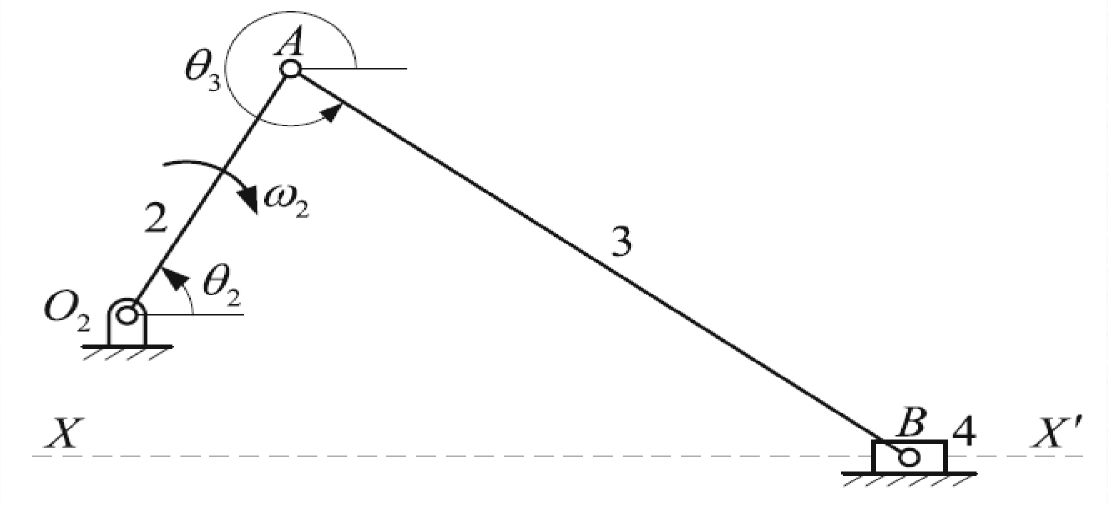
- Slider-crank linkage with constant angular velocity in link 2

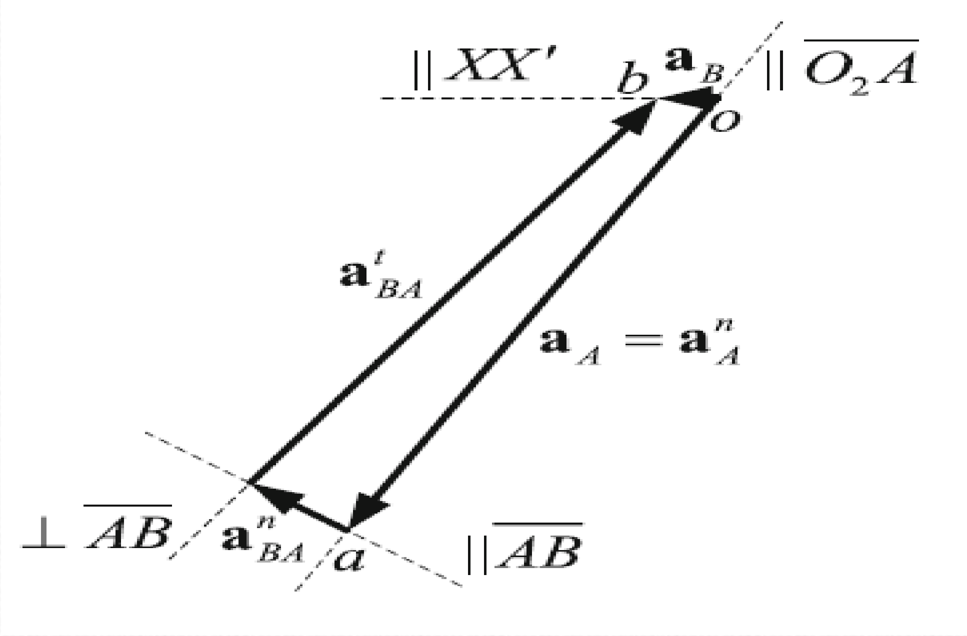


$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{BA} = (\mathbf{a}_A^n + \mathbf{a}_A^t) + (\mathbf{a}_{BA}^n + \mathbf{a}_{BA}^t)$$

Accelerations in a Slider-crank Linkage

- Acceleration polygon of the slider-crank . linkage in the Fig



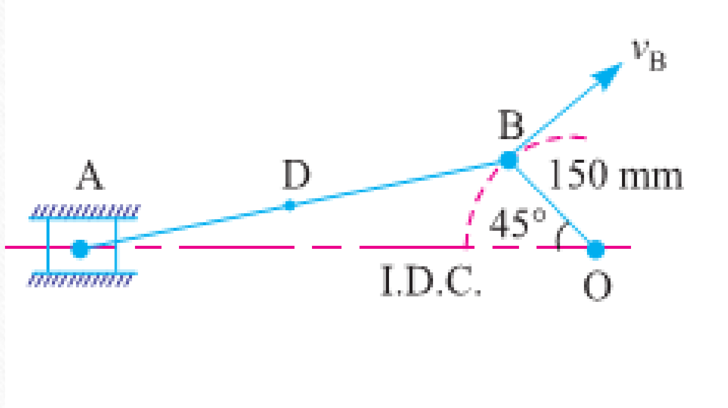


Accelerations in a Slider-crank Linkage

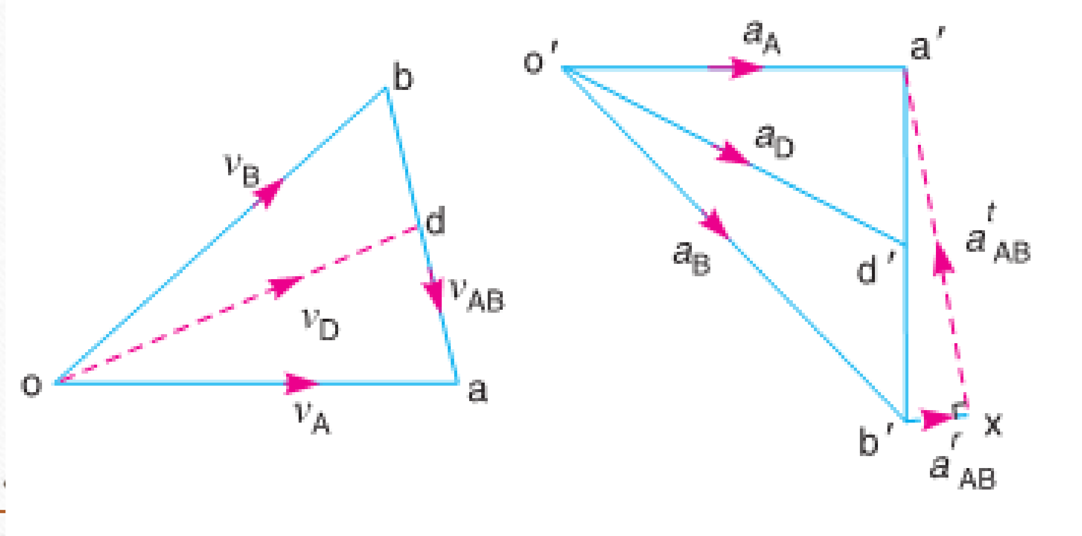
- *The crank of a slider crank mechanism rotates clockwise at a* ***constant*** *speed of 300 r.p.m. The crank is 150 mm and the connecting rod is 600 mm long. Determine :*

1. linear velocity and acceleration of the midpoint of the connecting rod, and

2. angular velocity and angular acceleration of the connecting rod, at a crank angle of 45° from inner dead center position.

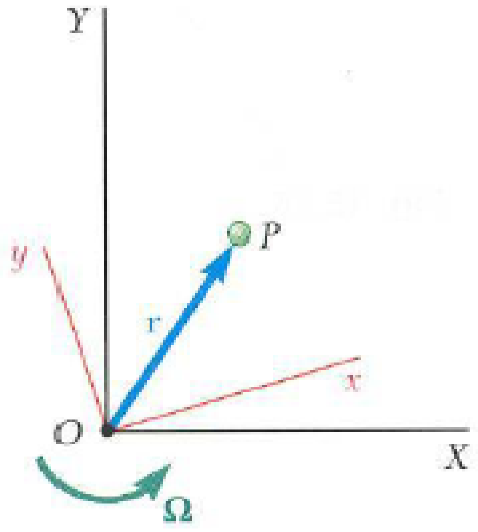


(a) Space diagram.



(b) Velocity diagram. (c) Acceleration diagram.

Coriolis Theorem



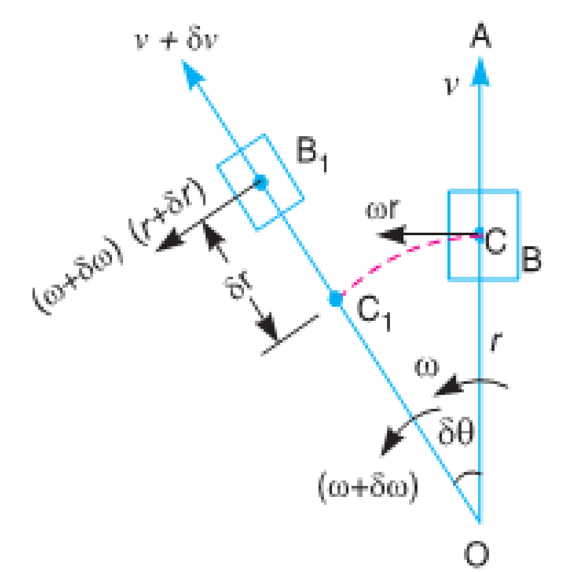
- Frame *OXY* is fixed and frame *Oxy* rotates with angular velocity $\vec{\Omega}$.
- Rate of change vector $\vec{r}_P$ for the particle *P* depends on the choice of frame.
- The absolute velocity of the particle *P* is

$$\vec{v}_P = \left(\dot{\vec{r}}\right)_{OXY} = \vec{\Omega}\times\vec{r} + \left(\dot{r}\right)_{Oxy}$$

Coriolis Theorem

- Consider a link *OA* and a slider *B* as shown in Fig. 8.26 (*a*). The slider *B* moves along the link *OA*. The point *C* is the coincident point on the link *OA*.

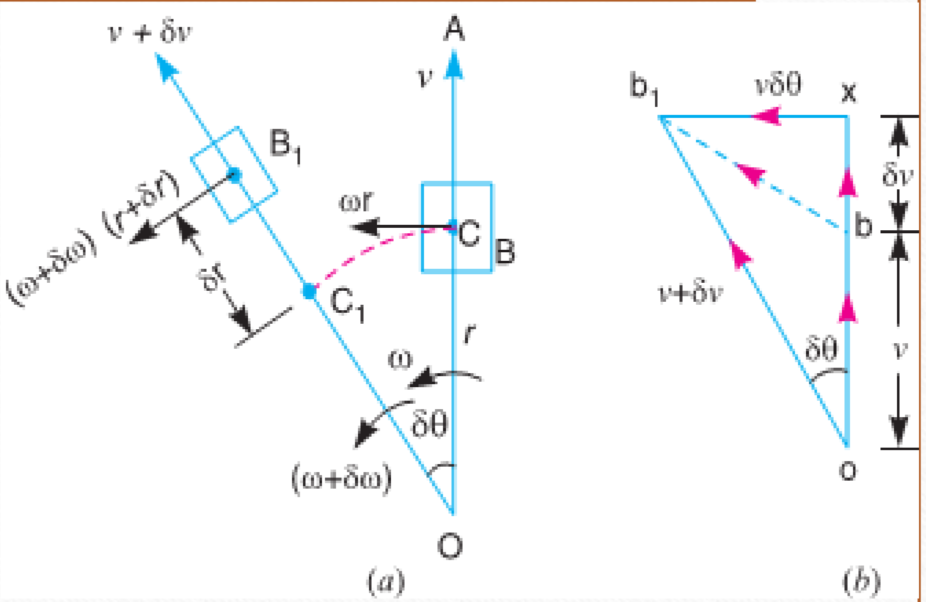
ω = Angular velocity of the link *OA* at time *t* seconds.
v = Velocity of the slider *B* along the link *OA* at time *t* seconds.
ω.*r* = Velocity of the slider *B* with respect to *O* (perpendicular to the link *OA*)
at time *t* seconds, and (ω + δω), (*v* + δ*v*) and (ω + δω) (*r* + δ*r*)



(*a*)

Coriolis Theorem

- Fig. 8.26 (*b*) shows the velocity diagram when their velocities v and $(v + \delta v)$ are considered.
  - In this diagram, the vector bb_1 represents the change in velocity in time δt sec ;
  - the vector bx represents the component of change of velocity bb_1 along OA (*i.e.* along radial direction) and
  - vector xb_1 represents the component of change of velocity bb_1 in a direction perpendicular to OA (*i.e.* in tangential direction).



$bx = ox - ob = (v + \delta v) \cos \delta\theta - v \uparrow$ δθ is very small $bx = (v + \delta v - v) \uparrow = \delta v \uparrow$ (Acting radially outwards)

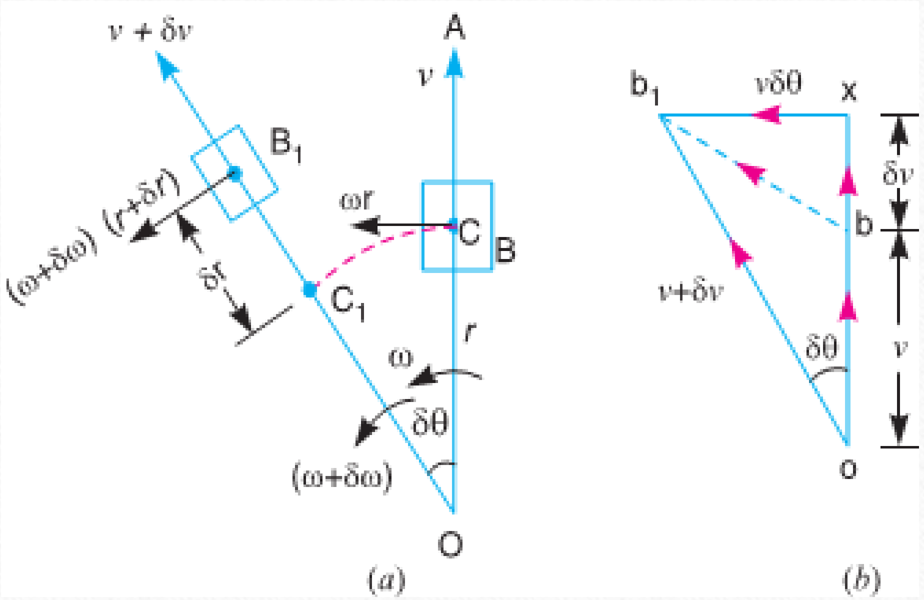
Coriolis Theorem

$xb_1 = (v + \delta v) \sin \delta\theta$ $\sin \delta\theta = \delta\theta$

$xb_1 = (v + \delta v)\, \delta\theta = v.\delta\theta + \delta v.\delta\theta$

Neglecting $\delta v.\delta\theta$ being very small

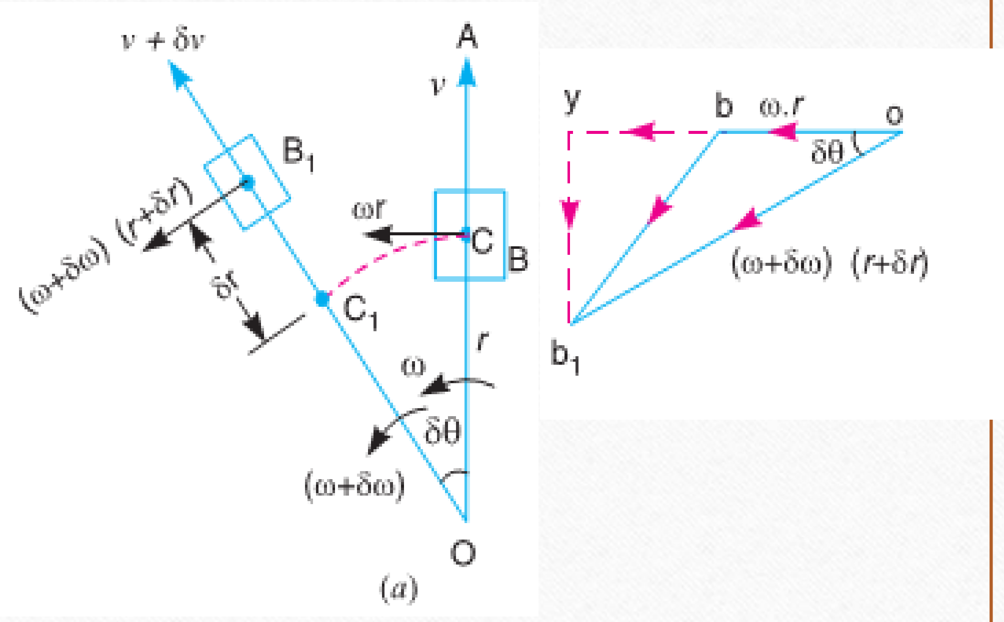
$\overleftarrow{xb_1 = v.\delta\theta}$ (Perpendicular to OA and towards left)



Coriolis Theorem

- Fig. 8.26 (*c*) shows the velocity diagram when the velocities $\omega.r$ and $(\omega + \delta\omega)(r + \delta r)$ are considered. In this diagram, vector *bb*1 represents the change in velocity ; vector *yb*1 represents the component of change of velocity *bb*1 along *OA* (*i.e.* along radial direction) and vector *by* represents the component of change of velocity *bb*1 in a direction perpendicular to *OA* (*i.e.* in a tangential direction)

$$yb_1 = (\omega + \delta\omega)(r + \delta r) \sin \delta\theta \downarrow$$
$$= (\omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r) \sin \delta\theta$$



Coriolis Theorem

$$yb_1 = \omega.r.\delta\theta + \omega.\delta r.\delta\theta + \delta\omega.r.\delta\theta + \delta\omega.\delta r.\delta\theta$$

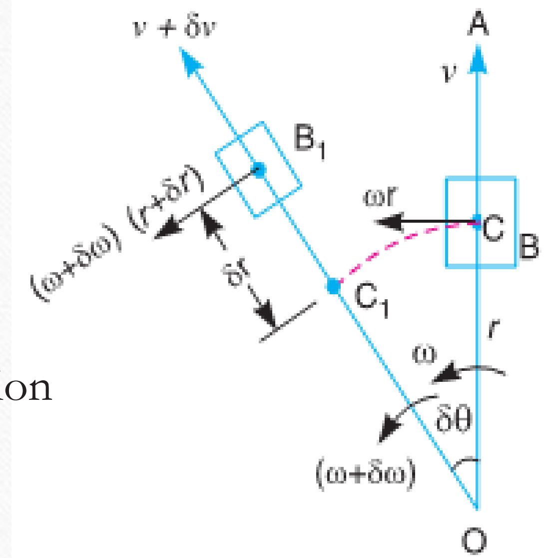
$$= \omega.r.\delta\theta \downarrow, \text{ acting radially inwards}$$

$$by = oy - ob = (\omega + \delta\omega)(r + \delta r)\cos\delta\theta - \omega.r$$

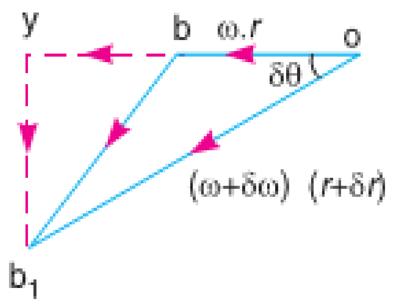
$$= (\omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r)\cos\delta\theta - \omega.r$$

Since $\delta\theta$ is small, therefore substituting $\cos\delta\theta = 1$, we have

$$by = \omega.r + \omega.\delta r + \delta\omega.r + \delta\omega.\delta r - \omega.r = \omega.\delta r + r.\delta\omega$$



(a)



Therefore, total component of change of velocity along radial direction

$$= bx - yb_1 = (\delta v - \omega.r.\delta\theta) \uparrow$$

∴ Radial component of the acceleration of the slider *B* with respect to *O* on the link *OA*, acting radially outwards from *O* to *A*,

$$a_{BO}^{r} = \text{Lt}\frac{\delta v - \omega.r.\delta\theta}{\delta t} = \frac{dv}{dt} - \omega.r \times \frac{d\theta}{dt} = \frac{dv}{dt} - \omega^2.r \uparrow \qquad (\because d\theta/dt = \omega)$$

Coriolis Theorem

The total component of change of velocity along tangential direction,

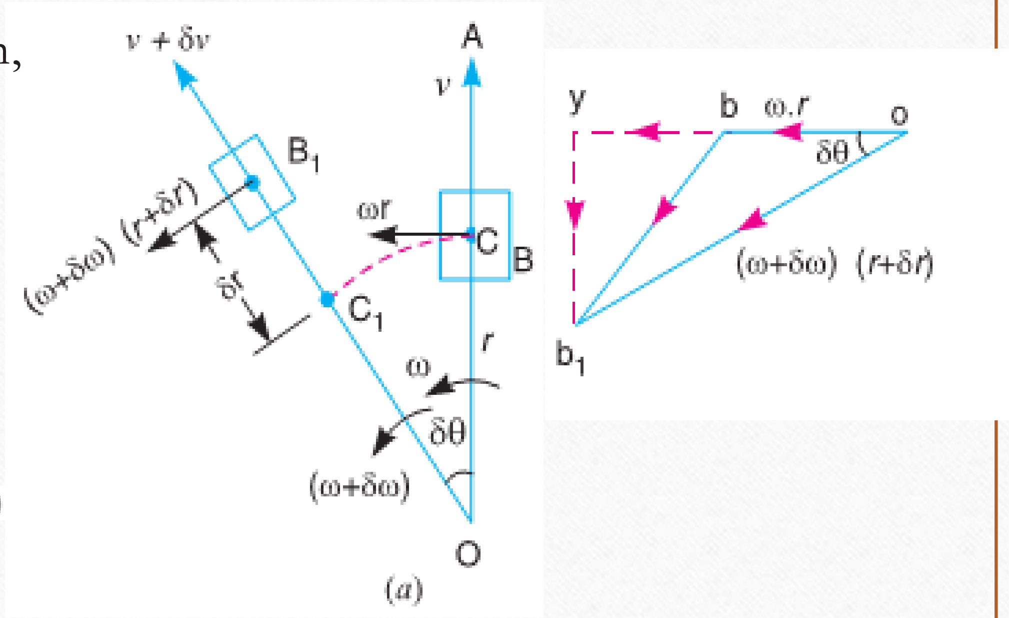
$$= xb_1 + by = v.\overset{\leftarrow}{\delta\,\theta} + (\omega.\overset{\leftarrow}{\delta r} + r.\delta\omega)$$

(Perpendicular to OA and towards left)

∴ Tangential component of acceleration of the slider B with respect to O on the link OA, acting perpendicular to OA and towards left,

$$a^t_{BO} = \text{Lt}\ \frac{v.\delta\theta + (\omega.\delta r + r.\delta\omega)}{\delta t} = v\frac{d\theta}{dt} + \omega\frac{dr}{dt} + r\frac{d\omega}{dt} \quad (\because\ dr/dt = v, \text{ and } d\omega/dt = \alpha)$$

$$= v.\omega + \omega.v + r.\alpha = (2\,\overset{\leftarrow}{v.\omega} + r.\alpha)$$



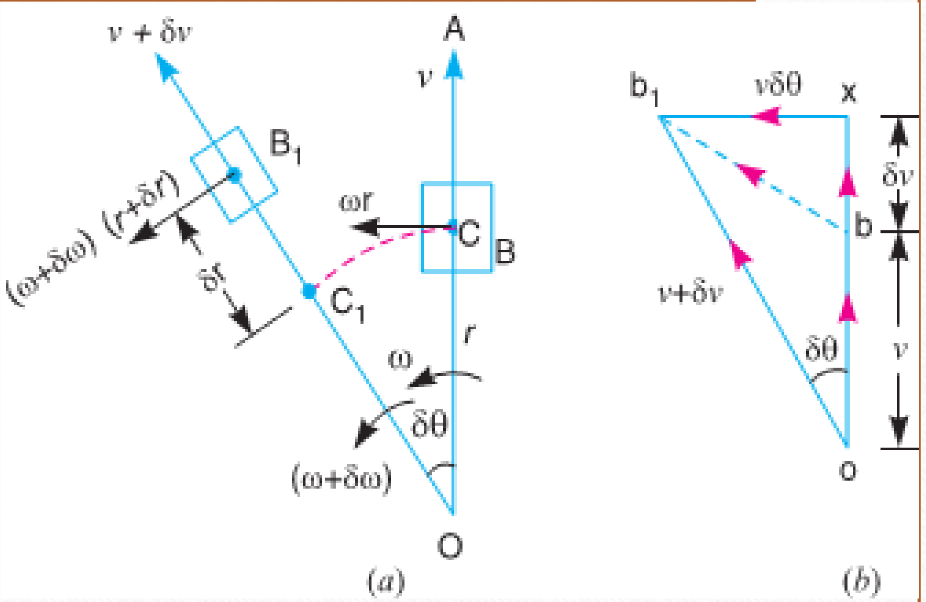
Coriolis Theorem

- Now radial component of acceleration of the **coincident** point *C* with respect to *O*, acting in a direction from *C* to *O*

$$a^r_{CO} = \omega^2 . r \uparrow$$

- Tangential component of acceleration of the **coincident** point *C* with respect to *O*, acting in a direction perpendicular to *CO* and towards left,

$$a^t_{CO} = \overleftarrow{\alpha . r} \uparrow$$



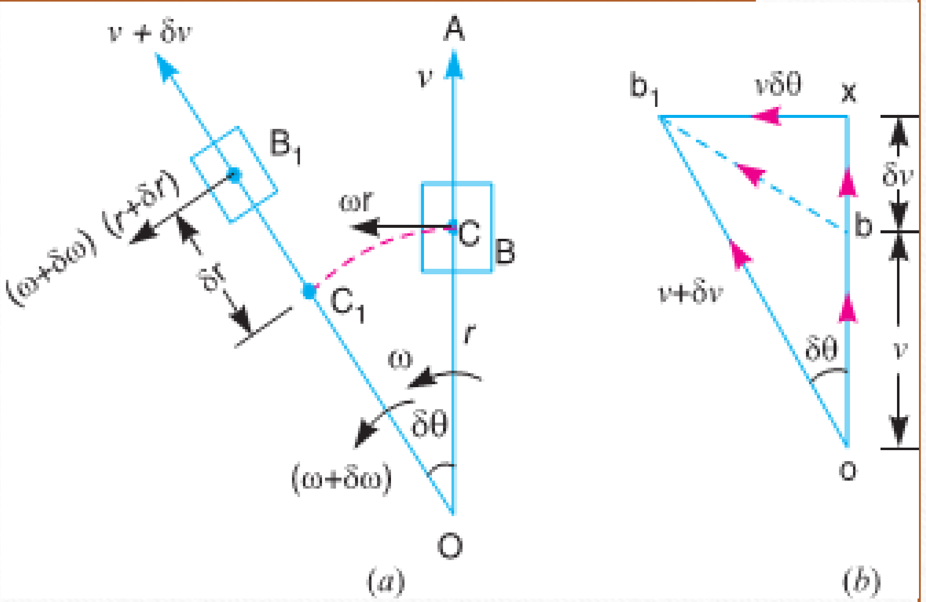
Coriolis Theorem

- Radial component of the **slider** *B* with respect to the coincident point *C* on the link *OA*, acting radially outwards

$$a^r_{BC} = a^r_{BO} - a^r_{CO} = \left(\frac{dv}{dt} - \omega^2.r\right) - \left(-\omega^2.r\right) = \frac{dv}{dt}\uparrow$$

- Tangential component of the **slider** *B* with respect to the coincident point *C* on the link *OA* acting in a direction perpendicular to *OA* and towards left,

$$a^t_{BC} = a^t_{BO} - a^t_{CO} = (2\,\omega v + \alpha.r) - \alpha.r = 2\overleftarrow{\omega v}$$



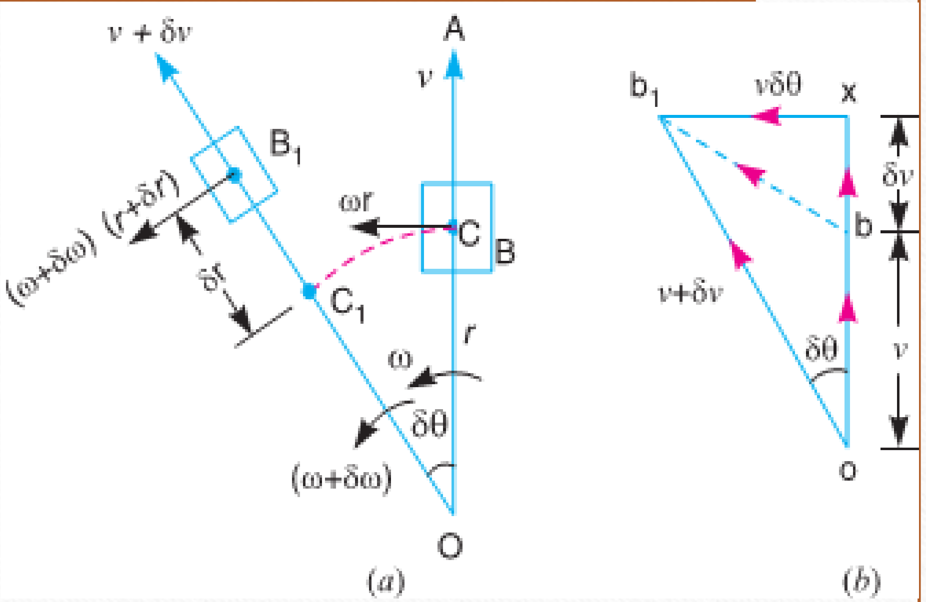
Coriolis Theorem

- This tangential component of acceleration of the slider *B* with respect to the coincident point *C* on the link is known as ***Coriolis component of acceleration*** **and is always perpendicular to the link.**
  ∴ Coriolis component of the acceleration of *B* with respect of *C*,

$$a^c_{BC} = a^t_{BC} = 2\omega.v$$

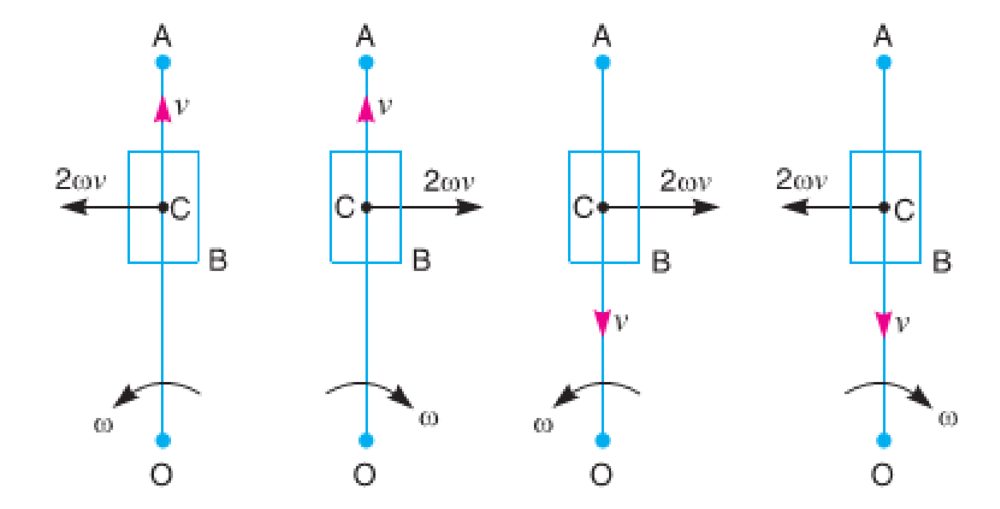
  - Where

ω = Angular velocity of the link *OA*, and
v = Velocity of slider *B* with respect to coincident point *C*.



Coriolis Theorem

- The anticlockwise direction for ω and the radially outward direction for *v* are taken as ***positive.***
- The direction of Coriolis component of acceleration will not be changed in sign if both ω and *v* are reversed in direction.



Coriolis Theorem

Example

- *A mechanism of a crank and slotted lever quick return motion is shown in Fig. 8.28. If the crank rotates counter clockwise at 120 r.p.m., determine for the configuration shown, the velocity and acceleration of the ram D. Also determine the angular acceleration of the slotted lever. Crank, AB = 150 mm ; Slotted arm, OC = 700 mm and link CD = 200 mm.*

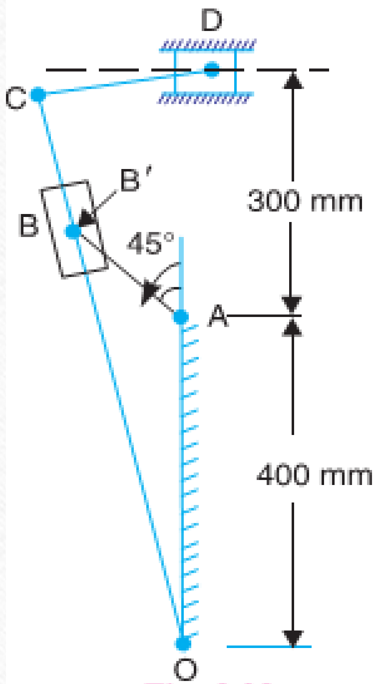


Fig. 8.28

Solution. Given : N_{BA} = 120 r.p.m or $\omega_{BA} = 2\pi \times 120/60 = 12.57$ rad/s ; *A B* = 150 mm = 0.15 m; *OC* = 700 mm = 0.7 m; *CD* = 200 mm = 0.2 m We know that velocity of *B* with respect to *A* ,

$$v_{BA} = \omega_{BA} \times AB$$

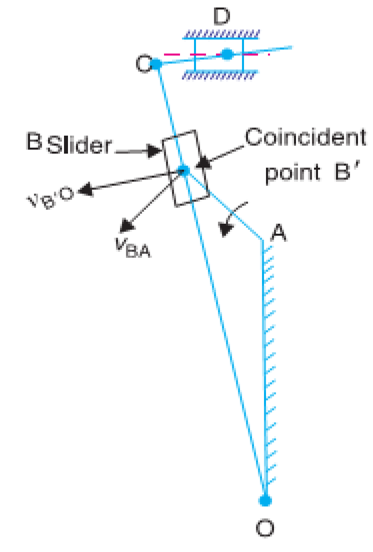
$$= 12.57 \times 0.15 = 1.9 \text{ m/s}$$

...(Perpendicular to *A B*)

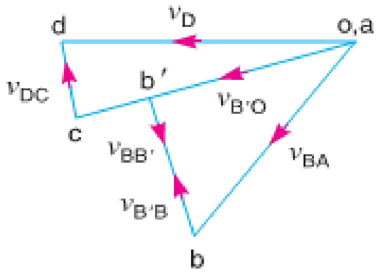
Coriolis Theorem

Example

- vector *ab* = *v*BA = 1.9 m/s
- Since the point *C* lies on *OB′* produced, therefore, divide vector *ob′* at *c* in the same ratio as *C* divides *OB′* in the space diagram. In other words, *ob’*/*oc* = *OB’*/*OC*
- Now from point *c*, draw vector *cd* perpendicular to *CD* to represent the velocity of *D* with respect to *C i.e.* *v*DC , and from point *o* draw vector *od* parallel to the path of motion of *D* (which is along the horizontal) to represent the velocity of *D i.e.* *v*D. The vectors *cd* and *od* intersect at *d*.
  By measurement, we find that velocity of the ram *D*,
- *v*D = vector *od* = 2.15 m/s



(*a*) Space diagram.

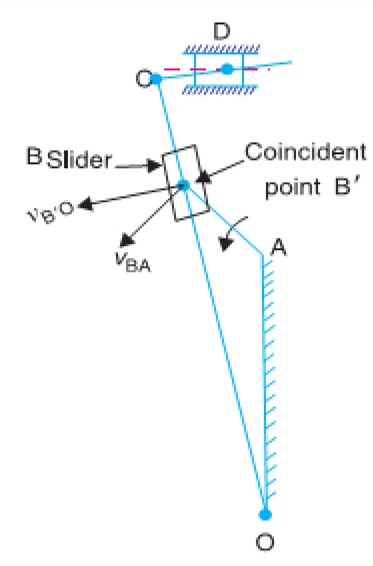


(*b*) Velocity diagram.

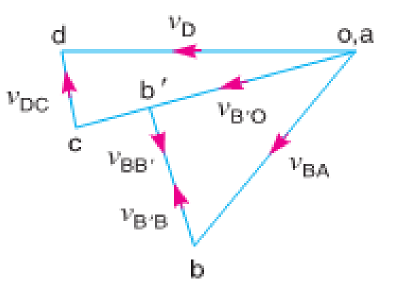
Coriolis Theorem

Example

- From velocity diagram, we also find that
  - Velocity of B with respect to B',
    $v_{BB'}$ = vector $b'b$ = 1.05 m/s
  - Velocity of D with respect to C,
    v_{DC} = vector cd = 0.45 m/s
  - Velocity of B' with respect to O
    $v_{B'O}$ = vector ob' = 1.55 m/s
  - Velocity of C with respect to O,
    v_{CO} = vector oc = 2.15 m/s
  - ∴ Angular velocity of the link OC or OB', $\omega_{CO} = \omega_{B'O} = \dfrac{v_{CO}}{OC} = \dfrac{2.15}{0.7} = 3.07$ rad/s (Anticlockwise)



(a) Space diagram.



(b) Velocity diagram.

Coriolis Theorem

Example

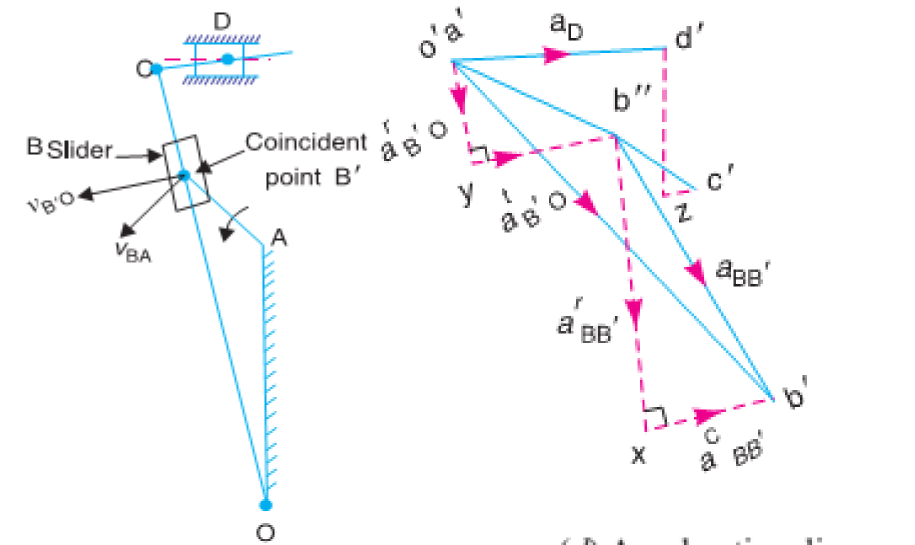
$$a^r_{BA} = \omega^2_{BA} \times AB = (12.57)^2 \times 0.15 = 23.7 \text{ m/s}^2$$

$$a^c_{BB'} = 2\omega.v = 2\omega_{CO}.v_{BB'} = 2 \times 3.07 \times 1.05 = 6.45 \text{ m/s}^2 \quad (\because \omega = \omega_{CO} \text{ and } v = v_{BB'})$$

$$a^r_{DC} = \frac{v^2_{DC}}{CD} = \frac{(0.45)^2}{0.2} = 1.01 \text{ m/s}^2$$

$$a^r_{B'O} = \frac{v^2_{B'O}}{B'O} = \frac{(1.55)^2}{0.52} = 4.62 \text{ m/s}^2 \quad \text{(By measurement } B'O = 0.52 \text{ m)}$$

These two components are mutually perpendicular. Therefore from point o', draw vector $o'y$ parallel to $B'O$ to represent $a^r_{B'O}$ = 4.62 m/s2 and from point y draw vector yb'' perpendicular to vector $o'y$ to represent $a^t_{B'O}$. The vectors xb'' and yb'' intersect at b''. Join $o'b''$. The vector $o'b''$ represents the acceleration of B' with respect to O, *i.e.* $a_{B'O}$.



(*a*) Space diagram.

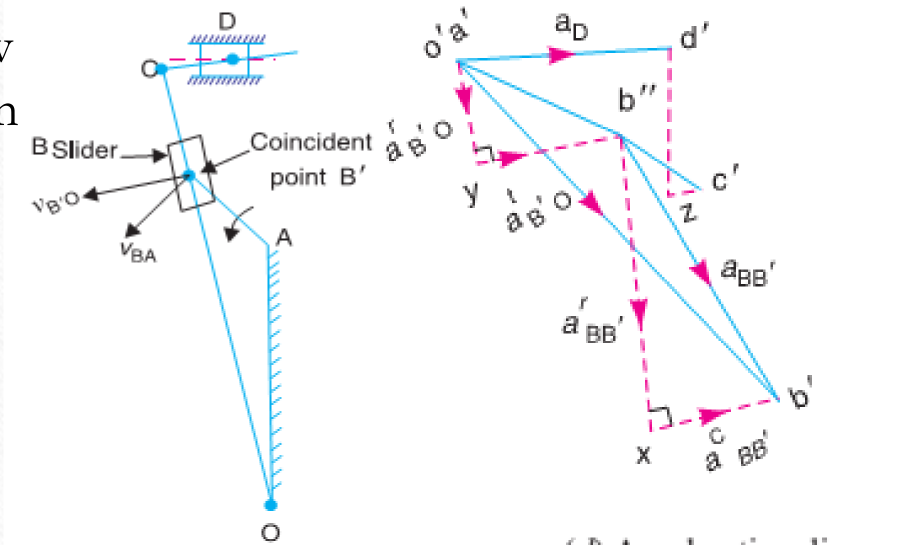
(*d*) Acceleration diagram.

Coriolis Theorem

Example

The two components are mutually perpendicular. Therefore draw vector $c'z$ parallel to CD to represent aDC^r = 1.01 m/s2 and from z draw zd' perpendicular to vector zc' to represent aDCt , whose magnitude is yet unknown.

From point o', draw vector $o'd'$ in the direction of motion of the ram D which is along the horizontal. The vectors zd' and $o'd'$ intersect at d'. The vector $o'd'$ represents the acceleration of ram D *i.e.* aD.



(*a*) Space diagram.

(*d*) Acceleration diagram.

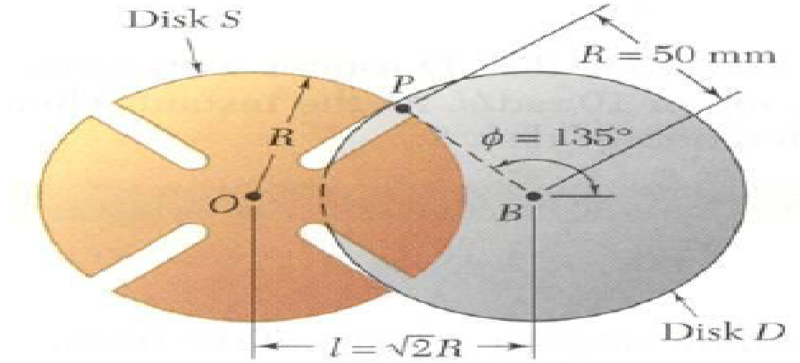
By measurement, we find that acceleration of the ram D,

$$a^t_{B'O} = \text{vector } yb'' = 6.4 \text{ m/s}^2$$

We know that angular acceleration of the slotted lever,

$$= \frac{a^t_{B'O}}{OB'} = \frac{6.4}{0.52} = 12.3 \text{ rad/s}^2 \text{ (Anticlockwise)}$$

Sample Problem



Disk D of the Geneva mechanism rotates with constant counterclockwise angular velocity $\omega_D = 10$ rad/s.

At the instant when $\phi = 150°$, determine (*a*) the angular velocity of disk *S*, and (*b*) the velocity of pin *P* relative to disk *S*.

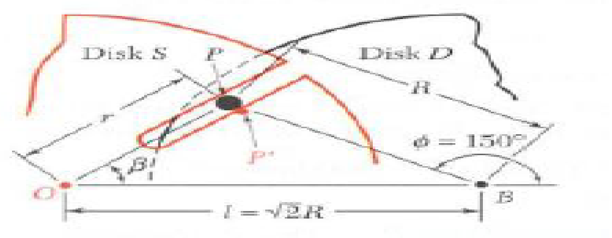
<u>SOLUTION</u>:

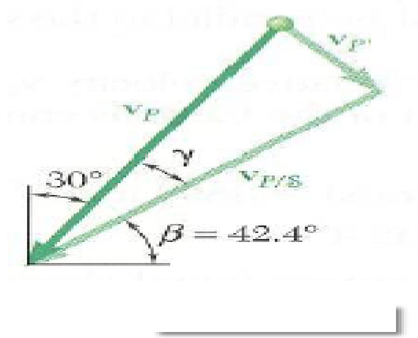
- The absolute velocity of the point *P* may be written as

  $$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/s}$$

- Magnitude and direction of velocity $\vec{v}_P$ of pin *P* are calculated from the radius and angular velocity of disk *D*.
- Direction of velocity $\vec{v}_{P'}$ of point *P'* on *S* coinciding with *P* is perpendicular to radius *OP*.
- Direction of velocity $\vec{v}_{P/s}$ of *P* with respect to *S* is parallel to the slot.
- Solve the vector triangle for the angular velocity of *S* and relative velocity of *P*.

Sample Problem





SOLUTION:

- The absolute velocity of the point P may be written as

  $$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/s}$$

- Magnitude and direction of absolute velocity of pin P are calculated from radius and angular velocity of disk D.

  $$v_P = R\omega_D = (50\ \text{mm})(10\ \text{rad/s}) = 500\,\text{mm/s}$$

- Direction of velocity of P with respect to S is parallel to slot. From the law of cosines,

  $$r^2 = R^2 + l^2 - 2Rl\cos 30° = 0.551R^2 \qquad r = 37.1\,\text{mm}$$

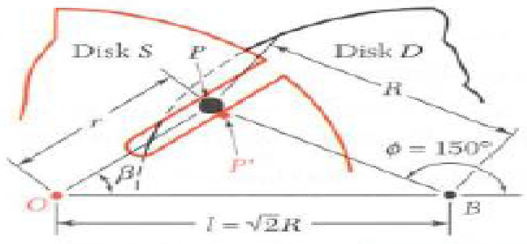
  From the law of cosines,

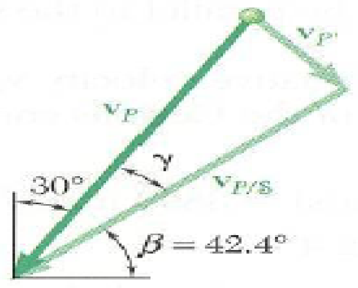
  $$\frac{\sin\beta}{\text{R}} = \frac{\sin 30°}{r} \qquad \sin\beta = \frac{\sin 30°}{0.742} \qquad \beta = 42.4°$$

  The interior angle of the vector triangle is

  $$\gamma = 90° - 42.4° - 30° = 17.6°$$

Sample Problem





$v_P = 500\,\text{mm/s}$

- Direction of velocity of point *P'* on *S* coinciding with *P* is perpendicular to radius *OP*. From the velocity triangle,

$$v_{P'} = v_P \sin\gamma = (500\,\text{mm/s})\sin 17.6° = 151.2\,\text{mm/s}$$

$$= r\omega_s \qquad \omega_s = \frac{151.2\,\text{mm/s}}{37.1\,\text{mm}}$$

$$\vec{\omega}_s = (-4.08\,\text{rad/s})\vec{k}$$

$$v_{P/s} = v_P \cos\gamma = (500\,\text{m/s})\cos 17.6°$$

$$\vec{v}_{P/s} = (477\,\text{m/s})(-\cos 42.4°\vec{i} - \sin 42.4°\vec{j})$$